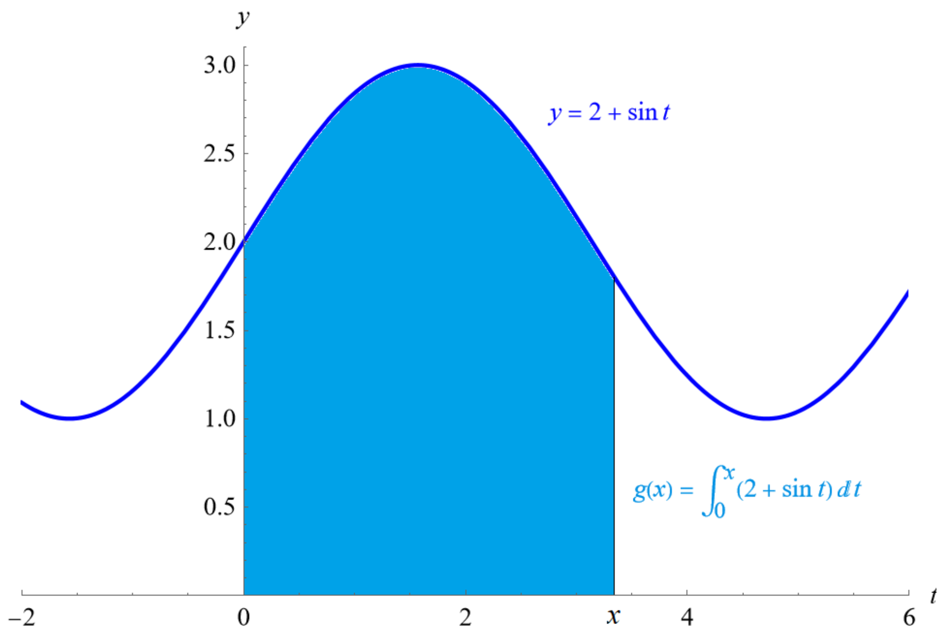
Exercise 6

Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

$$g(x) = \int_0^x (2 + \sin t)\, dt$$

Solution

$g(x)$ is the area under the curve $y = 2 + \sin t$ from $t = 0$ to $t = x$.



Part (a)

Get $g'(x)$ by taking the derivative of both sides and using part 1 of the fundamental theorem of calculus.

$$g'(x) = \frac{d}{dx}\int_0^x (2 + \sin t)\, dt$$

$$= 2 + \sin x$$

Part (b)

Evaluate the integral first using part 2 of the fundamental theorem of calculus

$$\begin{aligned} g(x) &= \int_0^x (2 + \sin t)\, dt \\ &= (2t - \cos t)\Big|_0^x \\ &= (2x - \cos x) - [2(0) - \cos 0] \\ &= (2x - \cos x) - (-1) \end{aligned}$$

and then take the derivative of both sides.

$$\begin{aligned} g'(x) &= \frac{d}{dx}(2x - \cos x + 1) \\ &= 2 + \sin x \end{aligned}$$

The same answer is obtained either way.